

# Optimal Unit Commitment Solution Methods

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**Abstract**— The Unit Commitment (UC) problem is a typical application of optimization methods to ensure an efficient, secure and economic operation of power systems. Its main objective is to determine online schedules and production levels for generating units, based on operational costs minimization, ensuring supply meets demand at all times. The process of finding an optimal schedule of generating units, subject to several technical constraints, given a planning horizon, has been solved by a diverse set of techniques. The formulations of the UC problem vary with energy systems characteristics, as well as with other economical, technical and environmental factors.

The present article mainly addresses the deterministic, single-objective Thermal UC problem. The formulation of the problem is discussed, and its solution is obtained by both exact and heuristic methods. Are studied and developed three of the most referenced methods in energy systems optimization: Dynamic Programming (DP), Lagrangian Relaxation (LR) and Particle Swarm Optimization (PSO). Both DP and LR are classical methods that have been shown to be very effective in the operational scheduling process. PSO is a more recent population based evolutionary algorithm that has been applied to various optimization problems, including UC. The effectiveness of the developed algorithms is tested on a 10-unit system case study. The obtained results show the better suitability of PSO, balancing a satisfactory solution with a decent computing time.

It is also proposed a model that integrates hydro-thermal and renewable units in an attempt show a glimpse of the real UC problem in the present day.

**Index Terms**— Unit Commitment, Dynamic Programming, Lagrangian Relaxation, Particle Swarm Optimization.

## I. INTRODUCTION

Energy systems are one of the most important infrastructures and one of the economic engines of a country, allowing its development and providing quality of life to its citizens. Energy is an everyday life essential asset for the modern societies and is of paramount importance to many companies, industry and the ordinary citizen. For electricity to be always available, the operation of the production system must be constantly planned. Worldwide demand for energy has been increasing at a pace that the expansion, planning and management of energy systems have become complex and challenging problems.

Energy systems can be divided into three main subsystems: Generation, Transmission and Distribution of energy. Each subsystem has its own behavioral characteristics and constraints that govern the operation of the overall system. The need to

provide electricity to consumers with the utmost safety and reliability obliges producer companies to plan energy supply processes at all levels. From the generation phase to the power supply to the final consumer, there are many economic considerations to consider. Thus, the planning steps should allow a reliable operation of the system while being economically sustainable.

The power system total load varies at every instant, so, the electric power companies must plan the power generation to meet this variable load in advance. To do this, they must decide between the available generators, which should be connected and when to synchronize them to the network, as well as the sequence in which the operating units should be turned off. This decision-making process is known as Unit Commitment (UC). By optimizing such decisions, energy can be produced at a lower cost, while satisfying demand and certain constraints. These restrictions may be of the system itself, which are used to ensure the power supply safety, or may be technological, reducing the freedom of choice between units, as well as the range of possible generation.

The traditional Unit Commitment problem is a single objective deterministic optimization problem. Its main objective is the operational scheduling of generating units using a criterion of minimization of the total operational costs during the time horizon considered, which could be several hours to a few weeks or years. The complexity of the problem depends on the diversity of the technological characteristics of energy systems and the generating units under consideration. The UC problem can be computationally challenging due to the high number of units present in a real energy system and the various constraints that each technology exhibits. For this reason, obtaining an optimal solution can be challenging.

The main objective of the present work is the study and analysis of some of the most popular methods applied to Unit Commitment resolution: Dynamic Programming (DP), Lagrangian Relaxation (LR) and Particle Swarm Optimization (PSO). The theoretical principles on which these methods are based will be analyzed, being also developed algorithms on MATLAB® that will serve to later apply to a 10-unit system case study. An analysis and comparison of the solutions obtained by each method, will allow to infer about the effectiveness of each one on the UC Problem resolution. The comparison between algorithms will allow to evaluate the possible evolution of performance of the most recent methods in relation to the older ones.

It is also the objective of this work to formulate the UC Problem of several generation technologies, to prove the increase of its complexity

## II. LITERATURE REVIEW

Unit Commitment is identified as a topic of enormous economic and technological relevance in the late 1950s, early 1960s. Until then, the scheduling and dispatch of thermoelectric units was performed through empirical techniques, typically with priority lists. These methods were far from optimal in economic terms, which translated into high production costs and high energy prices practiced to the final consumer. The first studies produced, [1] and [2], concluded about the need of Unit Commitment in energy systems from an economic point of view. They discuss several aspects and solution of the scheduling procedures to formulate the Unit Commitment problem, defining production costs functions and start-up and shut-down costs. The defined economic operation criterion to minimize the cost function expected value can be obtained through a constant periodic analysis of an equation that obtains the best combination of units to allocate. The publication [2] introduces the UC problem application of DP: a major breakthrough in power systems optimization.

### A. Classical Unit Commitment

Dynamic Programming was introduced by Lowery [2] and its application to the UC problem has since then been studied by several authors. Pang et al. [3] presents a DP approach for thermal units scheduling for a period up to 48 hours, including start-up costs, spinning reserve and minimum up and down times constraints. [4] compares several types of Dynamic Programming, and the Priority List method. This comparison showed that different versions of DP allow to achieve significant reductions in costs.

Lagrangian Relaxation is proposed on [5] and [6]. The main processes involve solving the dual problem in an approximate way, ignoring load or reserve constraints. An iterative process finds the possible dual reserve solution, properly adjusting Lagrange multipliers. Finally, for the given viable dual reserve solution, a viable global solution is obtained by executing an Economic Dispatch (ED) to satisfy the energy balance equations. This method has been used for decades in the UC problem solving and still has a major importance in helping other more advanced methods [7].

Mixed Integer Programming (MIP) is presented by John A. Muckstadt *et al.* [8] for the simultaneous resolution of the thermoelectric UC and ED problems. A.I. Cohen *et al.* [9] present a Branch and Bound algorithm, which is a combinatorial optimization method that discards non-viable solutions across lower and upper boundaries. A Mixed Integer Linear Programming (MILP) technique is proposed [10], which can be used for regulated or deregulated markets. The algorithm also provides the marginal energy price according to the constraints of the system.

Genetic Algorithms (GA) are applied to the UC problem on the article [11]. GA are a class of Evolutionary Algorithms that use techniques inspired by evolutionary biology such as heredity, mutation, and crossover.

Particle Swarm Optimization method is presented to solve the UC problem on [12]. The method uses the particles information to control the mutation operation and is similar to civil society as a group of "leaders" influence the rest of the population in choosing a better decision.

The publication [13] present the integration of different methods on a single algorithm in a hope to benefit from the advantages of each one and thus improve the effectiveness of the UC problem resolution. The results confirmed that the use of hybrid algorithms achieves results, in certain cases, with greater efficiency than other single methods.

### B. Stochastic Unit Commitment

Article [14] present a probabilistic model to analyze the risk resulting from the load uncertainty, which translates into the probability of committing insufficient capacity to compensate for unit faults and not foreseen load variations. The presented model solves the problem considering the power plant stops random nature, problems in transmission lines as well as uncertainty in load forecasting.

### C. Security Constrained Unit Commitment

System safety must always be one of the most important aspects of power systems. The goal of minimizing production costs directly conflicts with the need to ensure the safe operation of an energy system. Several models and methods have been presented to solve the operational scheduling of units considering safety power system restrictions. In [15], John J. Shaw proposed a new technique to solve System Constrained Unit Commitment. The technique includes reserve requirements and power flow restrictions on transmission lines to ensure system security.

### D. Profit Based Unit Commitment

With the liberalization of energy markets, the goal of producing energy at a minimal cost has ceased to make sense, when there are other competing companies on the market. On [16], the formulation of the Profit Based UC problem is presented in a perspective based on the new reality of the markets, where one does not try to minimize the cost, but to maximize the profit, which makes sense on non-monopolized markets.

### E. Unit Commitment with Environmental Considerations

Article [17] analyze the problem with environmental considerations, namely, the gas emissions from thermal units. Fuel costs and emissions minimization are contradictory objectives, turning the problem into a multi-objective optimization whose solution is represented by Pareto fronts: compromise curves between polluting emissions and fuel costs.

### F. Unit Commitment with Several Technologies

Environmental concerns have considerably increased, a particularly key factor on the UC problem, due to pollutants emissions. It was concluded that to reduce emissions of these substances, it would be necessary to implement new ways of introducing renewable energy technologies, which have a small environmental impact. The article [18] present a strategy to integrate thermal, hydro and renewable energy units.

### G. Conclusions

The studies published during the last 50 years show a complexity increase on the UC problem. The growing complexity of the problem has been accompanied by a constant evolution of techniques used in its resolution. Nowadays, the most used methods are Evolutionary and MIP Algorithms.

### III. UNIT COMMITMENT PROBLEM

#### A. Problem Description

The Unit Commitment problem consists on deciding, in a set of  $N$  generation units, when each unit  $j \in N$  must supply energy or not over a predefined time horizon  $T$ . In addition, it is decided, for each unit in operation in the instant  $t \in T$  to the time horizon  $T$ , which is the energy  $P_{t,j}$  that it must produce. Therefore, the problem includes two types of decisions, which are limited by load restrictions and technological constraints. For a group of  $N$  generating units and a time horizon  $T$ , the total number of possible combinations are  $(2^N - 1)^T$  (for 10 units and 24 hours, there are  $1.73 \times 10^{72}$  possibilities). Since there often are multiple solutions that satisfy demand and certain requirements, it is necessary to define a performance measure to choose the best solution. Typically, the units scheduling is performed with the objective of minimizing the total costs of running them on energy production processes.

The energy demand that the units must match varies greatly throughout the day and throughout the year. In an electrical power system, the total system load is generally higher during the day and early evening, when industrial loads are high, and most people are awake. The load drops then during the late evening and early morning, when most of the population is asleep. In addition, the use of electricity has a weekly cycle, with the load being lower on weekend days than on weekdays and a seasonal cycle, with energy consumption being higher on the winter than on the summer.

As already mentioned, Unit Commitment is performed with the objective of minimizing the total costs of operation. There are three types of costs:  $C_j(P)$ , generation costs,  $SCost$ , start-up costs, and shutdown costs, which are usually included in start-up costs. The cost involved in an optimal scheduling is given by the minimization of the total costs for all planning periods, with  $U_{t,j}$  being the state (1-on and 0-off) of the generator  $j$  in period  $t$ :

$$\text{Min} \sum_{t=1}^T \sum_{j=1}^N [C_j(P_{t,j}) \times U_{t,j} + SCost_{t,j} \times (1 - U_{t-1,j}) \times U_{t,j}] \quad (1)$$

#### B. Production Costs

Generally, production costs are modeled as a quadratic function in relation to the production level. The generation cost function illustration is given in Figure 1 and its expression is as follows:

$$C_j(P_{t,j}) = a_j P_{t,j}^2 + b_j P_{t,j} + c_j \quad (2)$$

Where  $a_j$  [€/MWh<sup>2</sup>],  $b_j$  [€/MWh] e  $c_j$  [€/h] are the coefficients of unit  $j$ .

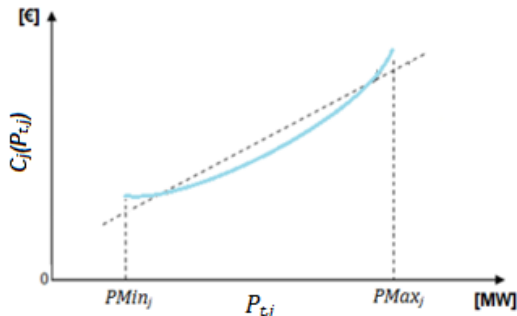


Fig. 1: Quadratic Function of Thermal Production Costs

#### C. Start-up Costs

Start-up costs are counted each time a generating unit is started and are often considered constant. However, on the steam turbine case, start-up costs should not be considered constant because they depend on the time the unit has been switched off and on the condition of the boiler, which may be hot or cold. If the boiler is kept warm during the period of inactivity, the start-up costs are usually modeled as a linear function over time:

$$SCost_t(t) = \alpha_j + \gamma_j \times HOFF_j(t) \quad (3)$$

Where  $HOFF$  is the number of hours the unit has been continuously off until the hour  $t$ ,  $\alpha_j$  [€] is the fixed start-up cost and  $\gamma_j$  [€/h] is the cost coefficient associated with fuel consumption to maintain the required temperature.

However, if the boiler is allowed to cool, the start-up costs are typically considered as exponentially time dependent, as in the following expression and Fig.2:

$$SCost(t) = \alpha + \beta \times (1 - e^{HOFF(t)/\tau}) \quad (4)$$

Where  $\beta$  (€) is the cold start-up cost and  $\tau$  is the cooling constant.

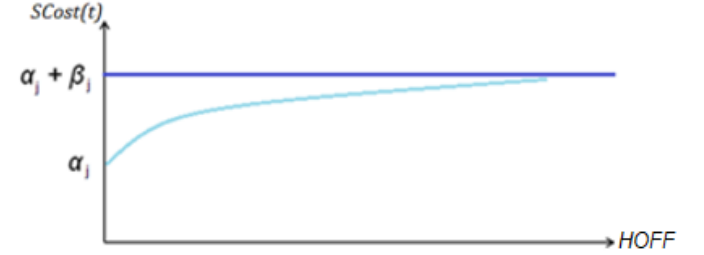


Fig.2: Cold start cost of a steam turbine generator

On the diesel groups case, start-up costs are more difficult to model as they can assume intermediate levels of heating and fuel changes. In general, a simplified cost model is used, which can be represented by:

$$SCost(t) = \begin{cases} SH, & \text{if } HOFF_{min} \leq HOFF(t) \leq HOFF_{min} + Hc \\ SC, & \text{if } HOFF(t) > HOFF_{min} + Hc \end{cases} \quad (5)$$

Where  $HOFF_{min}$  is the minimum idle hours required of the given unit,  $SH$  e  $SC$  are the hot and cold starting costs respectively, e  $Hc$  is a unit parameter such that  $HOFF_{min} + Hc$  indicates the number of hours the boiler needs to cool down.

#### D. Restrictions

Power systems must always meet customer demand  $D_t$ . It must also be ensured the ability to quickly generate additional energy, that is, a certain spinning reserve  $R_t$  should be committed. It should be noted that each unit has a certain minimum,  $PMin_j$ , and maximum,  $PMax_j$ , production limits.

##### Load Demand Restrictions:

$$\sum_{j=1}^N P_j \times U_j = D_t \quad (6)$$

##### Spinning Reserve Restrictions:

$$\sum_{j=1}^N PMin_j \times U_{t,j} \leq D_t + R_t \leq \sum_{j=1}^N PMax_j \times U_{t,j} \quad (7)$$

Generating units impose other restrictions on their characteristics and physical constraints. This includes unit capacity and production variance (positive:  $\Delta Up_j$  or negative:  $\Delta Down_j$ ) or minimum number of hours that the unit should be in each state: 1-ON ( $HONmin_j$ ) or 0-OFF ( $HOFFmin_j$ ).

**Production Range Restrictions:**

$$PMin_j \times U_{t,j} \leq P_{t,j} \leq PMax_j \times U_{t,j} \quad (8)$$

**Ramp Restrictions:**

$$-\Delta Down_j \leq P_{t,j} - P_{t-1,j} \leq \Delta Up_j \quad (9)$$

**Minimum Uptime and Downtime Constraints:**

$$HON_j(t) \geq HONmin_j \text{ and } HOFF_j(t) \geq HOFFmin_j \quad (10)$$

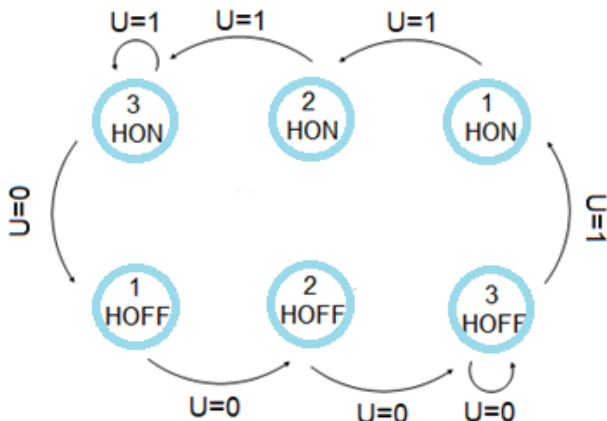


Fig. 3: Example of state transitions with minimum time constraints ( $HONmin=3h$  e  $HOFFmin=3h$ )

Further restrictions may be considered as being specific to certain generating units or resulting from the different production technologies that characterize energy systems.

**Must Run Restriction:** Some units receive mandatory execution status during certain times of the year for voltage support on the transmission network or other purposes such as steam supply that can be availed for other applications.

**Fuel Restrictions:** Some units may have limited fuel, or restrictions that require the burning of a specified amount of fuel at a given time.

**Hydro Restrictions:** The Thermal Unit Commitment cannot be completely separated from the scheduling of hydroelectric units.

#### IV. DYNAMIC PROGRAMMING

Facing the UC problem dimensionality issues and the inefficiency of Priority Lists, by itself, in solving the problem for most cases, several authors have proposed the application of Dynamic Programming to its resolution. Dynamic Programming has many advantages over Brute Force techniques of complete enumeration, the main one being a possible dimensionality reduction of the problem without sacrificing obtaining an optimal solution.

In general, Dynamic Programming is a recursive optimization method, making a sequence of interconnected decisions:

1. Defines a small part of the problem, finding an optimal solution for this part;
2. It slightly expands this small part of the problem, finding the optimal solution for the new problem using the optimal solution previously found;
3. Continues process 2 until the expansion of the problem leads to a problem that encompasses the fullness of the original problem. With this problem solved, the stopping conditions are satisfied;
4. The problem solution is constructed from the optimal solutions found for the small problems solved throughout the process.

To avoid a complete enumeration of all states, two variables can be added to the algorithm, n and m:

- n = number of states to search in each period.
- m = number of strategies or paths to save at each step.

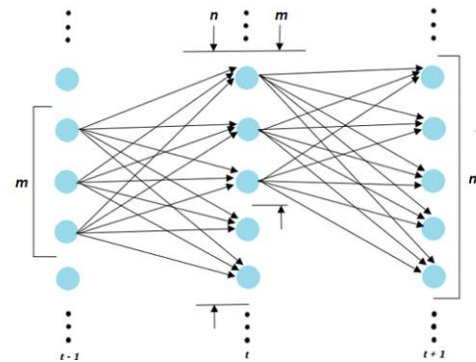


Fig. 4: Restricted search path in the Dynamic Programming algorithm ( $m=3$  and  $n=5$ ) [19]

#### A. Algorithm

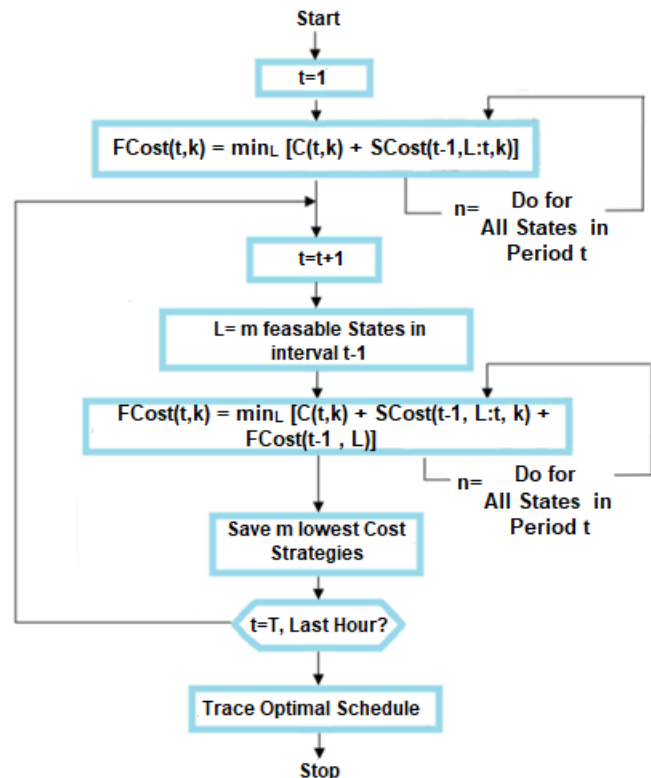


Fig. 5: Dynamic Programming algorithm flow chart [19]

### B. Objective Function

The Dynamic Programming recursive equation applied to the UC problem, at the time  $t$  for a state  $k$ , can be defined as:

$$FCost(t,k) = \min_L [C(t,k) + SCost(t-1, L; t, k) + FCost(t-1, L)] \quad (11)$$

Where:  $FCost$  is the lesser cumulative cost until state  $(t,k)$ ;  $SCost(t-1, L; t, k)$  is the start-up cost of the transition from the state  $(t-1, L)$  to  $(t,k)$ ;  $C$  is the production cost of the state  $(t,k)$ .

Given that the energy production cost function considered in this work is quadratic, the lossless economic dispatch will be computed through quadratic programming, by MATLAB function, *quadprog*. As the name implies, this "special" function minimizes the production costs quadratic function by analyzing different system and unit variables. The *quadprog* function provides the optimal output that each unit must generate to match the demand for a given combination. The function is presented in [20].

After obtaining the optimal production of each unit for a certain combination and hourly demand, the minimum production cost is obtained by adding the production cost quadratic functions, for  $P_j$ , of all units committed.

For a certain hourly demand, the process of finding the combination with the lowest cost of production can be a complicated task because of the many variables on which it depends and the various combinations of possible states. This process will be carried out through an increasing ordering function, with the first element being the least cost combination.

To incorporate start-up costs into the DP algorithm, when a unit changes from "0 state" to "state 1", it is sufficient to store the previous unit state in memory and compare it with the state of the combination to be considered:

- If the difference between the current state and the previous state is greater than zero, it means that the unit has been turned on in the current period and the unit starting cost in question must be added.
- If not, the unit has been switched off or it is on for at least two consecutive hours, with no additional costs.

### V. LAGRANGIAN RELAXATION

The great resulting benefits from using Lagrangian Relaxation are the decomposition of the problem, where each unit becomes a single entity, being optimized individually. Thus, the commitment of each unit is done optimally, but independently of the others. The main advantage of this method is achieved due to the load restrictions relaxation, not requiring the generated power to be equal to the demand at all the iterations. A possible solution is obtained by the iterative update of Lagrange multipliers, which approaches the relaxed solution, called the dual solution, to the solution that respects the load restrictions: the so-called primal solution.

The Lagrangian function is formed in the same way as in the Economic Dispatch problem resolution [19]:

$$\mathcal{L} = \sum_{t=1}^T \sum_{j=1}^N [C_j(P_{tj}) + SCost_{tj}] \times U_{tj} + \sum_{t=1}^T \lambda_t D_t - \sum_{t=1}^T \sum_{j=1}^N \lambda_t P_{tj} U_{tj} \quad (12)$$

Where  $\mathcal{L}$  is the Lagrangian and  $\lambda$  is the Lagrange multiplier.

### A. Dual Optimization Process

In the LR method, the coupling constraints are temporarily ignored, solving the problem as if they did not exist. This is done through the dual optimization procedure, which tries to reach the optimum by maximizing the Lagrangian in relation to the Lagrange multipliers (13), while minimizing it in relation to the other variables in the problem (14).

$$q^*(\lambda) = \max_{\lambda} q(\lambda) \quad (13)$$

$$q(\lambda) = \min_{P_j, U_{tj}} \mathcal{L}(P, U, \lambda) \quad (14)$$

The minimum of the function (14) is found for a certain optimal generation,  $P_{optj}$ , through its first derivative.

There are three distinct cases to keep in mind, depending on the  $P_{optj}$  relation with the generating units limits:

If  $P_{optj} \leq PMin_j$ , so:

$$\min [C_j(P_j) - \lambda_t P_{tj}] = C_j(PMin_j) - \lambda_t PMin_j \quad (15.a)$$

If  $PMIn_j \leq P_{optj} \leq PMax_j$ , so:

$$\min [C_j(P_j) - \lambda_t P_{tj}] = C_j(P_{optj}) - \lambda_t P_{optj} \quad (15.b)$$

If  $P_{optj} \geq PMax_j$ , so:

$$\min [C_j(P_j) - \lambda_t P_{tj}] = C_j(PMax_j) - \lambda_t PMax_j \quad (15.c)$$

It should be noted that, in order to minimize  $q(\lambda)$  in each state and that when the state  $U_{tj}=0$  this value is zero, then the only way to get a lower value is by having:

$$[C_j(P_j) - \lambda_t P_{tj}] < 0 \quad (16)$$

### B. Algorithm

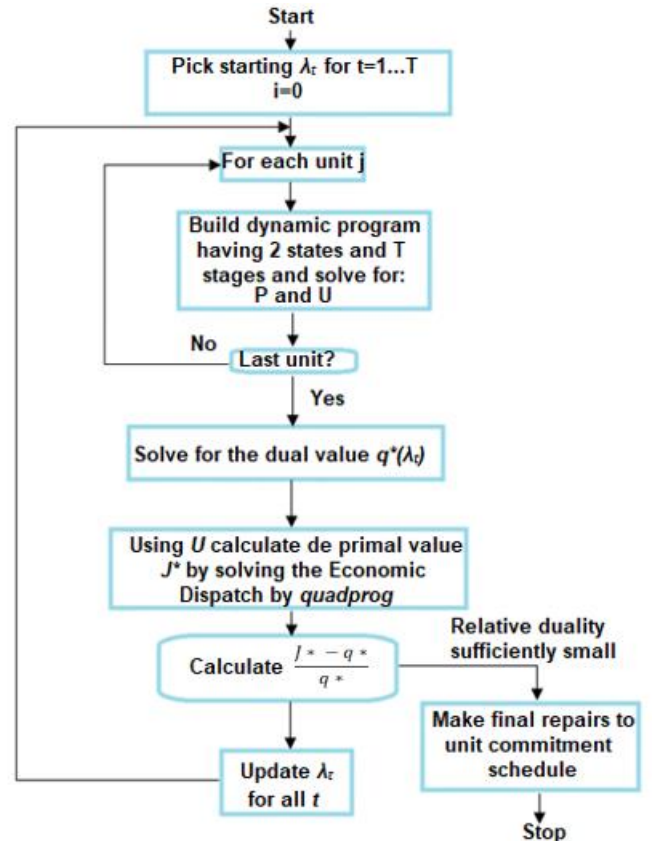


Fig. 6: Lagrangian Relaxation algorithm flow chart [19]

### C. Duality Gap

The difference between the primal and dual solutions is called duality gap, which must be minimized by approaching the dual value to the primal value.

The interval between the maximization of the dual cost functions ( $q^*$ ) and the minimization of the primal cost functions ( $J^*$ ), called the relative duality gap ( $\Delta$ ), can be used as a stop criterion. Therefore, the duality gap is used as a measure of convergence, being given by the following expression:

$$\Delta = \frac{J^* - q^*}{q^*} \quad (17)$$

### D. Adjusting $\lambda$

Throughout this work, the Lagrange multipliers adjustment will be done as follows, as proposed in [19]:

$$\lambda_t = \lambda_t + \left[ \frac{d}{d\lambda} q(\lambda) \right] \varepsilon \quad (18)$$

$$\varepsilon = 0.01 \quad \text{When} \quad \frac{d}{d\lambda} q(\lambda) > 0 \quad (19.a)$$

$$\varepsilon = 0.002 \quad \text{When} \quad \frac{d}{d\lambda} q(\lambda) < 0 \quad (19.b)$$

## VI. PARTICLE SWARM OPTIMIZATION

The particle swarm simulates a type of social optimization where each proposed solution (particle) is evaluated through an eligibility function, with the best leading the rest through the iterative process. This process is initiated to iteratively improve the candidate solutions, with the particles evaluating the suitability of these solutions, remembering the location where they had the best success. Each particle provides this location to adjacent particles, which also have access to the location where their neighbors have been most successful. The movements through the search space are guided by these successes, with the population converging, generally, to the best solution of the problem (or near it), at the end of the process.

Initially, the particles can randomly move in the search space, shifting according to the velocities derived from their current position and that of other particles in the swarm. The objective function of the problem is solved using each particle, and its assessed value is called *fitness*. Particles cross the entire search space with their own experience and that of others, with experience meaning their best physical value.

The best individual *fitness* value is stored as *pbest* (personal best) and the best of all particles is stored as *gbest* (global best). The social interaction between particles causes them to learn from each other and motivate them to move to better positions. So, the next movement of a particle, in a certain iteration, is motivated by experience, with its new velocity being influenced by its better physical *fitness* and that of its neighbors.

### A. Procedures

The PSO algorithm steps based on a fully connected network will be discussed below:

1. Randomly initialize the swarm of dimension equal to  $E$  particles, with a particle  $p_k$  on the position  $x_k^1$  of the search space, in iteration 1;
2. Compute the *fitness* value of each particle  $fitness(x^i)$ ;

3. Compare *fitness* of each particle their best *pbest* so far. This process can be defined as follows:

$$\text{If } fitness(x_k^i) < pbest_k^i, \text{Then: } \begin{cases} pbest_k^i = fitness(x_k^i) \\ x_{k,pbest}^i = x_k^i \end{cases} \quad (20)$$

4. Compare *fitness* of all particles to find *gbest*, according to the following expression:

$$\text{If } fitness(x_k^i) < gbest^i, \text{Then: } \begin{cases} gbest^i = fitness(x_k^i) \\ x_{gbest}^i = x_k^i \end{cases} \quad (21)$$

5. Update the velocity of each particle to the next iteration, where  $w$  is the weight of inertia,  $c1$  and  $c2$  are constants and  $rand$  is a random variable that assumes values between 0 and 1 in a uniformly distributed form:

$$v_k^{i+1} = w \times v_k^i + c_1 \times rand \times (x_{k,pbest}^i - x_k^i) + c_2 \times rand \times (x_{k,pbest}^i - x_k^i) \quad (22)$$

$$\text{Where: } w = w_{max} - \frac{w_{max} - w_{min}}{i_{max}} \times i \quad (23)$$

6. Move each particle to a new position:

$$x_k^{i+1} = \begin{cases} 0 & \text{se } rand() \geq sigmoid(v_k^{i+1}) \\ 1 & \text{se } rand() < sigmoid(v_k^{i+1}) \end{cases} \quad (24)$$

With:

7. Repeat from step 2 until desired convergence is achieved.

### B. Algorithm

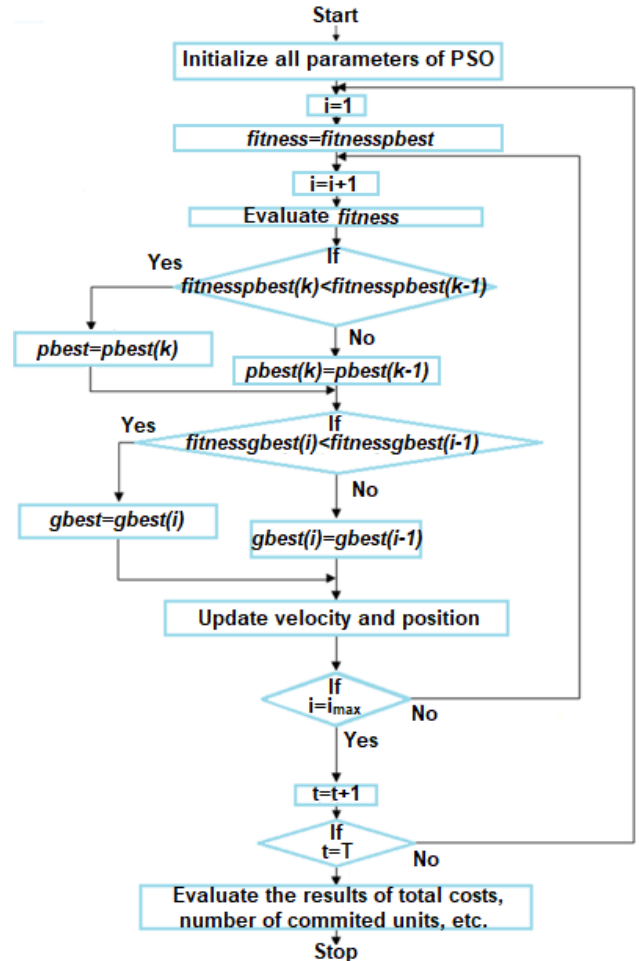


Fig. 7: PSO algorithm flow chart [21]

## VII. CASE STUDY

The case study that will serve as a test for the developed algorithms consists of a daily schedule (24 hours) of 10 units, the characteristics of which are found in the Appendix, Table A. The demand to which the generating units must meet follows a typical daily load profile whose values are presented in Table 1.

TABLE 1  
LOAD DEMAND ON THE CASE STUDY IN [MW]

Hour $t$	1	2	3	4	5	6
$D_t$	600	500	500	700	750	850
Hour $t$	7	8	9	10	11	12
$D_t$	900	1000	1050	1100	1200	1200
Hour $t$	13	14	15	16	17	18
$D_t$	1000	900	800	700	900	1100
Hour $t$	19	20	21	22	23	24
$D_t$	1300	1500	1600	1400	1000	800

### A. Application of DP

For this case study, the developed algorithm takes  $t_{comp}=1760$  seconds (almost 30 minutes) to get a final solution with a total operational cost of  $C_{Total}=451251\text{€}$ . The Fig.8 shows the hourly costs evolution as well as the evolution of the load demand versus the combined maximum power available,  $\sum_j^N P_{Max_j} \times U_j$ .

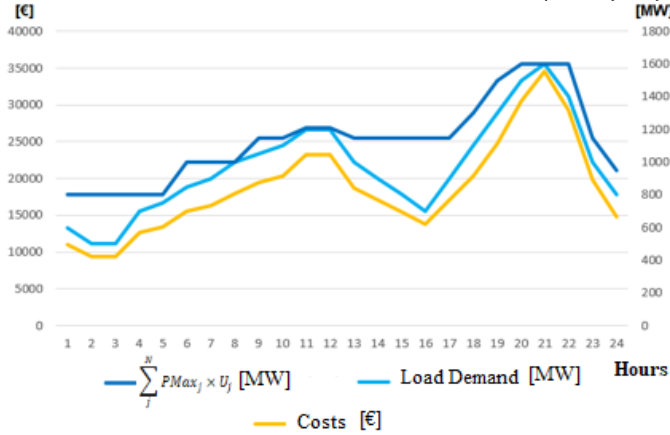


Fig.8: Hourly Production Costs and Load Demand versus Combined Maximum Power by DP

Evaluating the obtained results, it can be verified that the obtained combinations agree with the considered restrictions. In the first hours, being the load demand minimum, only the base units supply energy, since they have greater capacity of generation and a lower cost of production. Essentially, the base units stay connected to the network throughout the scheduling because they have the lowest production cost and because they must be connected for many hours. As demand increases, the most economical available units will be initialized, which should stay on for a few hours. However, in certain cases it is convenient, in a small momentary energy spike, to initialize a more expensive unit rather than a more economical one, if this more expensive unit has no running time restrictions and has a low start-up cost. This happens at hours 11 and 12, with the algorithm opting 2 hours for the unit 8, to the detriment of several cheaper units that were available to be initialized but have more restricted conditions.

### B. Application of LR

The algorithm reaches the stop conditions,  $\Delta < 0.06$ , after 17 iterations, with about 4 seconds of computation time and a total operational cost of  $C_{Total}=459996\text{€}$ .

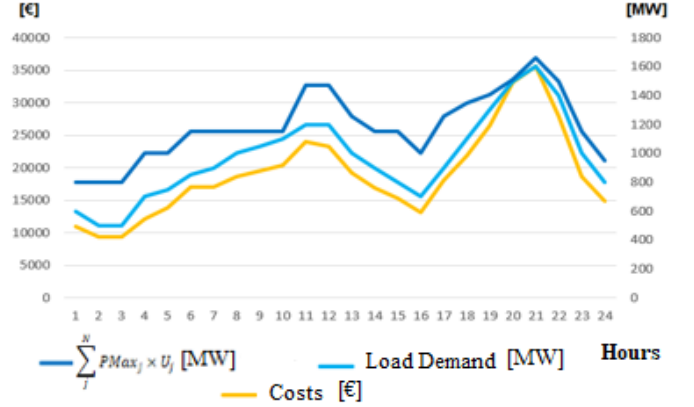


Fig.9: Hourly Production Costs and Load Demand versus Combined Maximum Power by LR

The iterative evolution of the primal  $J^*$  and dual  $q^*$  solutions as well as of  $\Delta$  can be verified in Figure 10. The iterative evolution of  $\lambda_t$  is represented by Figure 11.

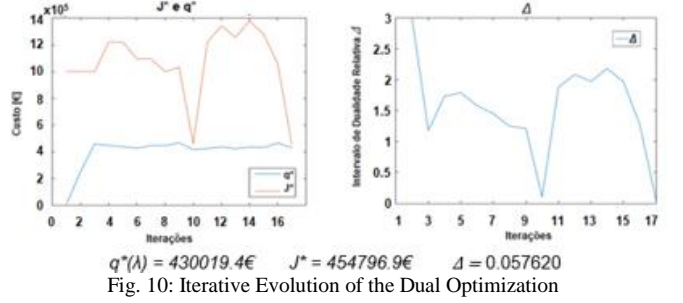


Fig. 10: Iterative Evolution of the Dual Optimization

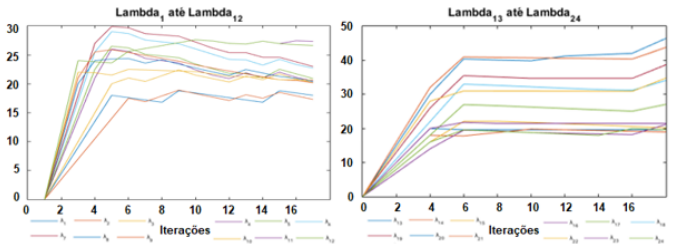


Fig. 11: Iterative Evolution of  $\lambda_t$

The final solution presents an excess of reserve in most hours which contrasts with the solution obtained by Dynamic Programming. For that reason, the total cost is much higher. As can be seen in Figure 10, the duality gap becomes quite small as the dual optimization proceeds. The values of  $\lambda_t$  greatly increase in the first 3 or 4 iterations, which translates into a rapid growth of the  $q^*$  solution.  $\lambda_t$  then stabilize which causes the stabilization of  $q^*$ . Convergence is unstable near the end of the process, meaning that some units are "switched on" and "off", which causes instability in the final solution. Primal solution  $J^*$  is initially defined as a much higher value (1 million €) than the expected solution value and is only minimized when the dual solution provides a schedule that meets demand at all times and respects all constraints.

### C. Application of PSO

The characteristic PSO parameters used in this problem will be  $E=50$ ,  $i_{max}=10$ ,  $w_{max}=0.9$  e  $w_{min}=0.4$ . The computational execution of the ten iterations took 50 seconds with a total operational cost of  $C_{Total}=452510\text{€}$ .

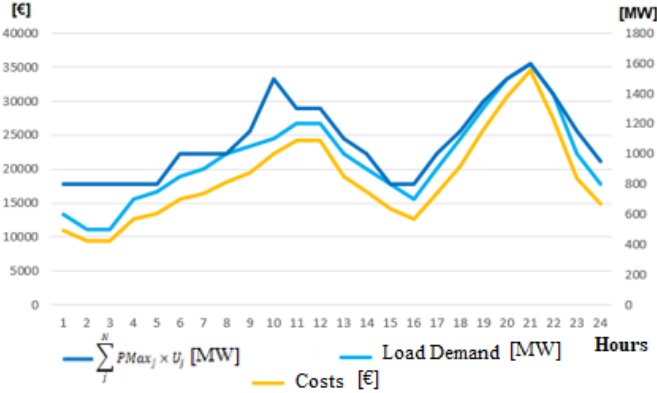


Fig. 12: Hourly Production Costs and Load Demand versus Combined Maximum Power by PSO

Fig. 13 shows the iterative evolution of the total cost of operation,  $g_{best}$ .

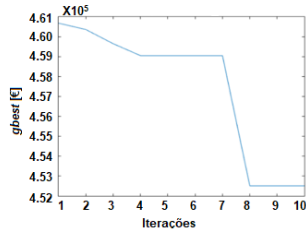


Fig. 13: iterative evolution  $g_{best}$ .

For a relatively complex case study like the present, obtaining a satisfactory solution is not as fast (iteratively) as it is for a simple case with only a few viable solutions. In cases with many units ( $\geq 10$  generators), there are possibly many solutions that respect all problem constraints. On the random process of particle creation, there is hardly one that represents the optimal solution at the first iteration. In theory, the mechanisms that govern the PSO nature enhance an iterative improvement of the solution, translating into a decrease of the  $g_{best}$  value until the final iteration. For this case, the solution obtained by the PSO algorithm,  $g_{best}$ , is improved up to iteration 8, as the particles move to less expensive solutions.

### D. Comparison of Algorithms

Figure 14 compares the hourly costs of the final solution obtained by each method: DP, LR and PSO.

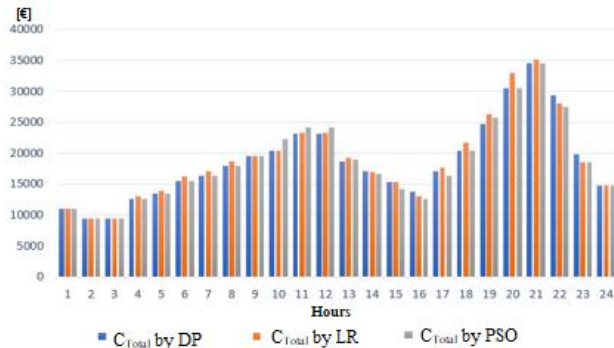


Fig. 14: Comparison of Hourly Costs for each method

Table 2 presents, for each method, the computation time, the total cost and its relative percentage difference in relation to the optimal solution cost.

TABLE 2  
COMPARATIVE DATA BETWEEN ALGORITHMS

	DP	LR	PSO
$t_{comp}$ [s]	1766s	4.29	49.7
$C_{Total}$ [€]	451251	459996	452510
$(C_{Total} - C_{Total}^{Opt})/C_{Total}^{Opt}$ [%]	0	1.94	0.28

### VIII. WIND AND HYDRO-THERMAL UNIT COMMITMENT

The increased environmental considerations led to the incorporation of renewable energy technologies in production systems. The use of renewable energy, such as wind energy (through wind turbines), as well as hydropower (through hydroelectric plants), has the great advantage of having a small environmental impact when compared with thermal energy production technologies. Hydroelectric power has been used for more than a century and is a technology that is relatively developed, with its market share stabilized (except for some fluctuations inherent to the various levels of precipitation on each year). The use of new renewable energy technologies on electricity production has been increasingly implemented. In the last two decades, these alternative sources of energy production have gained prominence in the market in several countries of the world, a trend that will become even more pronounced soon.

#### A. Problem Description

The Thermal, Hydro and Wind UC problem can be solved using a method that combines Lagrangian Relaxation, with Sequential Unit Commitment (SUC) and Unit Decommittment (UD) [22]. It can be assumed that, at a certain time, wind energy is not subject to dispatch, having priority access to the grid. Knowing the hourly wind generation of energy,  $P_{wind_t}$ , through forecasting methods, it is enough to commit the hydro and thermal units, as shown on the flowchart of Fig.15. The hydrothermal units will only have to meet a load equal to the hourly demand less the generation that the wind turbines will supply (25). This simplification can be done because, if the wind power units are able to generate energy (i.e. wind speed is sufficient to be profitable to produce energy), all this energy will have priority, being definitively supplied to the grid (except losses of energy). It should also be assured that the possible maximum hydrothermal production is greater than the demand and reserve minus the wind generation.

$$\sum_{j=1}^N P_{t,j} \times U_{t,j} + \sum_{h=1}^H (Ph_{t,h} \times U_{t,h} - P_{pump_{t,h}}) + P_{wind_t} = D_t \quad (25)$$

$$\sum_{j=1}^N P_{Max_j} \times U_{t,j} + \sum_{h=1}^H (Ph_{Max_h} \times U_{t,h} - P_{pump_{t,h}}) \geq D_t + R_t - P_{wind_t} \quad (26)$$

$$P_{wind_t} = \varphi_{w_t} \times P_{wind_{Max}} \quad (27)$$

With:  $Ph_h$  being the production of a  $h$  hydro unit,  $P_{pump}$  the energy spent on water pumping and  $Ph_{Max}$  being the maximum production of a hydro unit.  $P_{wind_{Max}}$  is the maximum wind production and  $\varphi_{w_t}$  is the [%] of availability of wind on hour  $t$ .



## B. Algorithm of Hydro-Thermal UC

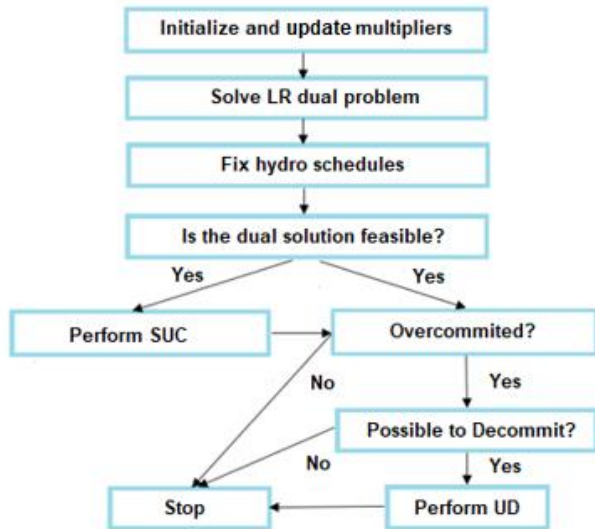


Fig.15: Hydro-Thermal UC Flowchart through LR, SUC and UD [22]

## IX. CONCLUSIONS

It can be concluded that all developed algorithms solve the Unit Commitment Problem successfully. Each of the studied methods has advantages and disadvantages in their UC problem applications:

- **Dynamic Programming:** It has the advantage of reaching an optimal solution. Despite this, it presents a dimensionality problem that manifest itself on a high computation time for real systems.
- **Lagrangian Relaxation:** Achieves a viable solution very quickly, even for complex systems. Its disadvantage is the poor economical solution sometimes achieved.
- **Particle Swarm Optimization:** It is the method that best balances a solution close to the optimum, with a good computation time. However, it rarely achieves an optimal solution.

Nowadays, we face many challenges on the electricity generation field, due to the growing focus on renewable energies, which have great ecological advantages. There have been major changes in energy systems, modifying their operational paradigms. The commitment to increase the electricity production from renewable energies, characterized by great variability (and unpredictability), must be incorporated in the Unit Commitment Problem, increasing, however, its complexity of resolution.

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## APPENDIX

TABLE A

DATA OF THE GENERATING UNITS USED ON THE CASY STUDY

<i>Unit</i> <i>j</i>	<i>Pmin<sub>j</sub></i> [MW]	<i>PMax<sub>j</sub></i> [MW]	<i>a<sub>j</sub></i> [€/MW <sup>2</sup> ]	<i>b<sub>j</sub></i> [€/MW]	<i>c<sub>j</sub></i> [€/h]	<i>SCost<sub>j</sub></i> [€]	<i>HONmin<sub>j</sub></i> [h]	<i>HOFFmin<sub>j</sub></i> [h]	<i>Ini. State</i> [h]
1	150	400	0.0005	15	800	4000	7	7	8 HON
2	150	400	0.0003	16	900	3500	7	7	5 HON
3	50	200	0.002	16	500	1000	5	5	2 HOFF
4	40	150	0.002	17	650	800	5	4	5 HOFF
5	40	150	0.003	20	450	500	5	5	6 HOFF
6	35	110	0.007	23	350	200	3	3	5 HOFF
7	30	90	0.008	26	440	200	3	3	5 HOFF
8	20	60	0.005	25	660	150	1	1	5 HOFF
9	10	50	0.002	27	650	100	1	1	6 HOFF
10	10	50	0.0015	28	600	100	1	1	6 HOFF